BEAL'S CONJECTURE AS GLOBAL BREAK-THROUGH IN NATURAL SCIENCES*

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Abstract

The validity of Beal's Conjecture in mathematics is determined by arithmetic geometry methods known yet in ancient times. It turns out that this theorem plays the essential role in cognition of objective world.

Keywords: Beal's Conjecture, Fermat's Last Theorem, fractal geometry, quantum approach, global science.

"You will know the truth, and the truth will make you free"

The gospel of John 8:32

1. Introduction. Beal's Conjecture as generalized Fermat's Last Theorem.

Beal's Conjecture [1] is in fact a singular counterexample to the famous proof of Fermat's Last Theorem finally issued in 1995 [2]. Indeed, the Beal proposition deals with arbitrary powers of whole numbers combined in one equation similarly to the well-known equation of Fermat's Last Theorem but it has never been proved by the methods produced in [2]. On the contrary, the Beal conjecture can be solved by the ancient Greek arithmetic geometry methods applied successfully also to the Fermat problem too [3].

However the author's original proof of Beal's Conjecture stumbled upon the wall of non-acceptance in high mathematical circles, although the work was professionally fulfilled in rigorous mathematical manner. His paper was rejected without consideration in such mathematical journals as *Notices of the AMS* and *Journal of Number Theory*. Then the author submitted it to *Eurasian Mathematical Journal* in March of this year but the decision of its Editorial Board has not been received for the time being. That is why the author decided to give here his solution of Beal's Conjecture as generalized Fermat's Last Theorem [1] adapted to conference's comprehension and abilities and exposing some new trends of global development in natural sciences.

2. Arithmetic geometry of Beal's Conjecture reveals the significance of Fermat's Last Theorem for global science.

The author's proof of Beal's Conjecture is related to the part of number theory defined as arithmetic algebraic geometry. But for the purpose of adopted exposition pure algebraic approaches and definitions will be avoided in order to remain just within the bounds of arithmetic methods of number theory.

Let us write the Beal conjecture equality in the following way:

$$x^n + y^n = z^n \tag{1}$$

with positive integers x, y, z having a common factor and exponent n taking simultaneously the next spectrum of values: $n = (k, l, m)$, where integers k, l, m at least 3 and n has one independent value for each term. Thus we assume at the beginning that equality (1) exists.

Then we can explore some arbitrary solutions of equation (1) in whole numbers. Consider equality (1) as a partition of z^n into two parts x^n and y^n written in whole numbers. It resembles the Pythagorean equation in real numbers. If we could reduce (1) to the degree 2 with whole parts in it, then one could easily make certain that partition (1) is perhaps true during checking up by transferring units from one part of the sum to the other as counters on a counting line. To produce such scaling, let us introduce the notion of right-angled numbers.

__Definition__. Right-angled number is such a non-negative real number, the square of which is a whole non-negative number.

The set of right-angled numbers **P** = {0, 1, √2 , √3 , 2, √5, …} is countable. The system of right-angled numbers P = ⟨**P**,+,·,0,1⟩ is defined by operations of addition and multiplication and two singled out elements (zero and unit). The system P is non-closed in relation to addition. Notice that the set of non-negative whole numbers is a subset of the set of right-angled numbers. Then consider (1) on the 2-dimensional lattice of right-angled numbers.

For this reason, one can rewrite (1) as an equality for some coprime *x'*, *y'*, *z'* and common whole factor *d* : $(x'd)^k + (y'd)^l = (z'd)^m$ and fulfill scaling-down: $(z'd)^2 = (x'd)^k / (z'd)^{m-2} + (y'd)^l / (z'd)^{m-2} = (x')^k d^{k-m+2} / (z')^{m-2} + (y')^l d^{l-m+2} / (z')^{m-2} = x_o^2 + y_o^2$, where x_o^2 and y_o^2 are squares of some right-angled numbers x_o and y_o .To get whole parts in the sum of equality (1), one must regard exponents $(k-m+2)$ and $(l-m+2)$ with base *d* equal to $(z')^{m-2}$. Obviously, *k* and *l* have to be more or equal $m-1$. If *k* or *l* does not satisfy this rule, then equality (1) cannot be represented on the lattice of right-angled numbers and consequently constructed in natural numbers. However, if $(k, l) \geq m-1$, equality (1) assumes the following character (quantic) after fulfilling scaling-up:

$$z^m = x^k + y^l = z^{m-2} (x_o^2 + y_o^2) \tag{2}$$

Now let us apply the ancient method of getting powers of whole numbers [9] and produce two chains of proportions connected with each other with some equality presenting integer *z* as a sum of two whole numbers:

$$\begin{aligned} z/x &= x/k = k/k_1 = \ldots = k_{m-3}/k_{m-2} \\ z/y &= y/l = l/l_1 = \ldots = l_{m-3}/l_{m-2} \end{aligned} \tag{3}$$

where *z, x, y* integers from (1), *m* natural index at least 3, and $z = k_{m-2} + l_{m-2}$, k_{m-2} and l_{m-2} some whole parts of *z* taken from the method of scaling-down (see lower).

From proportions (3) one can obtain the next formulae:

$$\begin{aligned} &x^2 = kz = (k_1z/x)z\,,\ x^3 = k_1z^2 = (k_2z/x)z^2\,,\ \ldots,\ x^m = k_{m-2}\,z^{m-1}, \\ &y^2 = lz = (l_1z/y)z\,,\ y^3 = l_1z^2 = (l_2z/y)z^2\,,\ \ldots,\ y^m = l_{m-2}\,z^{m-1}, \end{aligned} \tag{4}$$

and get $x^m = (zk_{m-2})z^{m-2}$, $y^m = (zl_{m-2})z^{m-2}$, where k_{m-2} and l_{m-2} are found from the basic equality (1):

$$z = (z'd) = (x'd)^k /(z'd)^{m-1} + (y'd)^l /(z'd)^{m-1} = k_{m-2} + l_{m-2}$$

Then exponents *k* and *l* have to be more or equal *m*, if k_{m-2} and l_{m-2} are to be whole with $d = (z')^{m-1}$ as a minimum.

Now count that $zk_{m-2} = x_o^2$, $zl_{m-2} = y_o^2$, where x_o , y_o are right-angled numbers from (2) when $d = (z')^{m-1}$, and get $x^m = x_o^2 z^{m-2}$, $y^m = y_o^2 z^{m-2}$. Hence square roots of x^m, y^m are proportional means between x_o^2 and z^{m-2}, y_o^2 and z^{m-2}. Furthermore, relations (4) give only one-valued powers in partition (1), i.e., $x^m = \boldsymbol{x}^k$, $y^m = \boldsymbol{y}^l$ (here we do not make distinctions between designations of like variables except contrast). Thus we equalized degrees *k* and *l* to *m* in the quantic (2) and got the following identity for the equal similar partitions of z^n into two whole parts:

$$z^m = x^m + y^m = z^{m-2}(x_o^2 + y_o^2) = x^k + y^l \tag{5}$$

where $x^k = (x^{k/m})^m = x^m$, $y^l = (y^{l/m})^m = y^m$, i.e., *k*, *l* cannot be more or less than *m* in order to satisfy boundaries of the right-angled lattice. Therefore $(k, l) = m$, since roots with degrees $m \geq 3$ cannot be numbers of the right-angled lattice and bases *x, y* may be only whole powers beginning with exponent 1 under *m*. In other words, *m* serves as a special quantifier for degrees of equation (1).

This yields that (1) comes to the Fermat equality in integers:

$$x^m + y^m = z^m,\ m \geq 3 \tag{6}$$

Then (6) can be reduced to the hypothetical equality in coprimes, which is impossible according to Fermat's Last Theorem. Now one can prove Fermat's Last Theorem with the same methods in order to fulfill solution of the Beal conjecture in full and one measure, especially as generally accepted proof of Fermat's Last Theorem [2] contains in itself contradiction in terms from the point of view of conventional set theory.

Indeed, according to [2] the "elliptic" curve *E* associated with Fermat's equation $a^l + b^l = c^l$ is given as the set of solutions $\{x, y\}$ for the next equation:

$$E:\ y^2 = x\,(x - a^l)\,(x - c^l)$$

supplemented with the neutral element ∞ ("an infinitely distant point") being actually an infinite set of solutions. But the set theory forbids using sets (including infinite ones) as their own elements. Therefore it is no wonder that the given assumption leads eventually to logical error of the type *"circulus vitiosus"* when the indicated curve *E* turns out to be an illusive elliptic curve, i.e., non-existent in linear topological space, from the very beginning of proof and not only in the end of it [3].

Thus true correct proof of Fermat's Last Theorem is needed to complete the solution of Beal's Conjecture.

Proof of Fermat's Last Theorem. Fermat's Last Theorem claims that the following equation (7) with integers *z, x, y* and natural exponent $n > 2$ has no solution:

$$z^n = x^n + y^n \tag{7}$$

Let us check this assertion. Suppose however that at least one solution was found. Then we shall try to construct such a solution and make certain of its impossibility. We shall work in the system of right-angled numbers (see above *Definition*).

Consider (7) on the 2-dimensional lattice of right-angled numbers with coordinates x_0 , y_0 and norm $z^2 = x_0^2 + y_0^2$ differing by its square fragments and being a partition of number z^2 into two summands represented by non-negative whole numbers. The minimal (non-zero) norm (standard) of right-angled numbers equals 1.

To construct powers of whole numbers presented in (7), let us produce two chains of continued proportions connected with each other by the norm $z^2 = x_0^2 + y_0^2$:

$$\begin{aligned} &z/x_0 = x_0/k = k/k_1 = \dots = k_{n-3}/k_{n-2} \\ &z/y_0 = y_0/l = l/l_1 = \dots = l_{n-3}/l_{n-2} \end{aligned} \tag{8}$$

where natural indices of the last terms of each chain in (8) are getting from $n > 2$. Continued proportions (2) yield the following formulae:

$$\begin{aligned} &kz = x_0^2,\ k_1z = x_0k,\ k_2z = x_0k_1,\ \dots,\ k_{n-2}z = x_0k_{n-3} \\ &lz = y_0^2,\ l_1z = y_0l,\ l_2z = y_0l_1,\ \dots,\ l_{n-2}z = y_0l_{n-3} \end{aligned} \tag{9}$$

$$\begin{aligned} &x_0^2 = kz = (k_1z/x_0)z,\quad x_0^3 = k_1z^2 = (k_2z/x_0)z^2,\ \dots,\ x_0^n = k_{n-2}z^{n-1} \\ &y_0^2 = lz = (l_1z/y_0)z,\quad y_0^3 = l_1z^2 = (l_2z/y_0)z^2,\ \dots,\ y_0^n = l_{n-2}z^{n-1} \end{aligned} \tag{10}$$

Now it is necessary to fix the norm for the partition of z^n into two like powers in (7). As in the case of Beal's Conjecture, let us assume that *z, x, y* in presupposed equality (7) have a common factor *d*, i. e., $z = (z'd)$, $x = (x'd)$, $y = (y'd)$, where *z', x'. y'* coprime. Thereupon we divide equality (7) by z^{n-1} and get:

$z = (z'd) = (x'd)^n/(z'd)^{n-1} + (y'd)^n/(z'd)^{n-1} = k + l$, where *k* and *l* integers if $d = (z')^{n-1}$ as a minimum. From this and (9)-(10) it follows that $z^2 = x_0^2 + y_0^2$ and $z^n = z^{n-2}(x_0^2 + y_0^2)$ is a scaled-up modification of the norm $z^2 = x_0^2 + y_0^2$.

Further, one can get a singular partition of z^n into three terms from (10) for the given norm when $n > 2$:

$$z^n = x_0^n + y_0^n + \lambda_n \tag{11}$$

where $\lambda_n = z^{n-1}[(k - k_{n-2}) + (l - l_{n-2})]$ is a remainder after subtracting x_0^n and y_0^n out of z^n such that $\lambda_n > 0$ when $n > 2$ and $x_0 y_0 \neq 0$, $\lambda_n = 0$ when $n = 2$ and $x_0 y_0 \neq 0$, $x_0, y_0 \in [0, z]$, $z \in (0, \infty)$.

Partition (11) can be reduced to the norm, from which it was obtained:

$$z^n = x_0^n + y_0^n + \lambda_n = z^{n-2} (x_0^2 + y_0^2) \tag{12}$$

Formula (12) represents by itself a combinatorial equality of two partitions in three and two terms. If it would not be so, equality (7) could not have the same norm and chains of proportions, from which it was obtained, would be different from (8). In the case of right-angled numbers this equality is realized only when x_0, y_0 integers.

Thus scaling invariance of the norm $z^{n-2} (x_0^2 + y_0^2)$ leads to the next equalities of different fragments of partitions (12):

$$x_0^n + y_0^n = (x^n \text{ or } y^n) \tag{13}$$

and correspondingly $\lambda_n = (y^n \text{ or } x^n)$. It can be noticed that $x_0^n \neq z^{n-2} \cdot y_0^2 = y^n$ and $y_0^n \neq z^{n-2} \cdot x_0^2 = x^n$ because of the lack of coincidence of decompositions in factorization of numbers x_0^n and y^n, y_0^n and x^n. Obviously, $x_0^n \neq z^{n-2} \cdot x_0^2$ and $y_0^n \neq z^{n-2} \cdot y_0^2$.

Let us show now that x_0 and y_0 cannot be irrational in (13) on account of integer partition of z^n into x^n and y^n. Here two cases can occur: when n is an odd number (designate it by $\nu = n_{\text{odd}} \geq 3$) and when n is an even number (designate it by $\mu = n_{\text{even}} \geq 4$). Considering the first case we find that x_0 and y_0 cannot be irrational in (13) as irrational square roots do not give a rational number in sum.

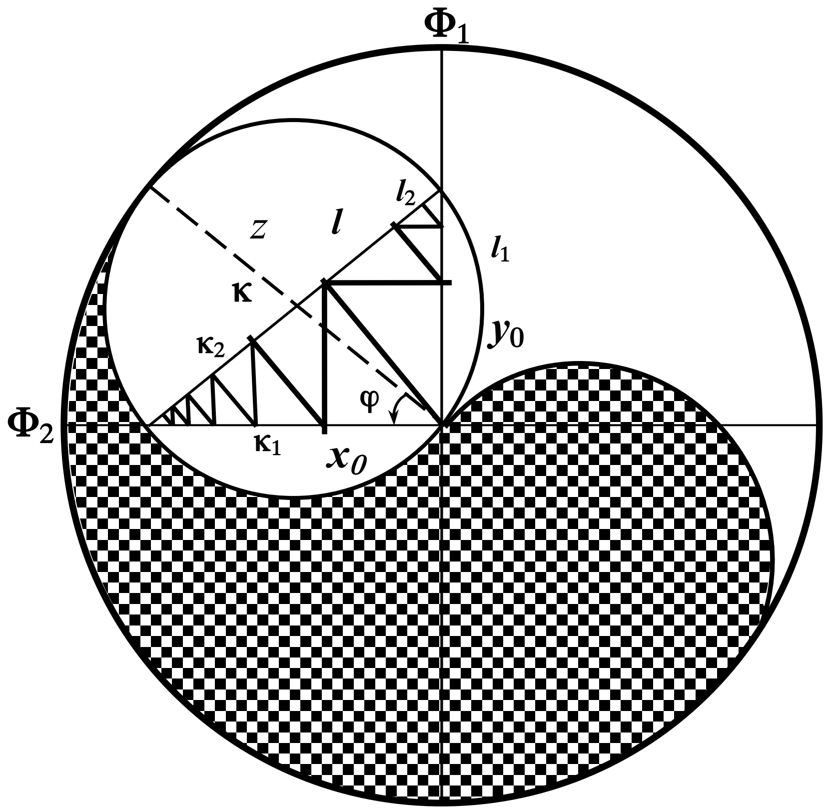
Let us consider the second case when $n = \mu$. Indeed, from the one hand, there is Pythagorean triple of numbers z^m, x^m, y^m with $m = \mu/2$ such that $(z^m)^2 = (x^m)^2 + (y^m)^2$. On the other hand, the initial equality can be written in the form $z^2 = x_0^2 + y_0^2$ showing that the indicated triple of numbers corresponds to the triple z, x_0, y_0 describing the like right-angled triangle. Therefore $z^m/x^m = z/x_0$, $z^m/y^m = z/y_0$, $x^m = x_0 \cdot z^{m-1}$, $y^m = y_0 \cdot z^{m-1}$ and x_0 and y_0 are not irrational.

So it was revealed, as a result of the previous calculation, that equality (13) consists of whole numbers. Furthermore, Fermat's triple obtained from them for the given $n > 2$, for example, x_0, y_0, x, is not the same by value as Fermat's triple x, y, z from (7), since $x_0 / y_0 \neq x / y$ that is clear from the following: $x_0^2/y_0^2 = x^n/y^n = (x^2/y^2)(x^{n-2}/y^{n-2})$.

Hence equality (13) reduced to the form (12) describes another right-angled triangle different from that defined by Pythagorean triple x_0, y_0, z.

Let us come back to the assumption at the beginning of the proof that integer solution (7) exists. This assumption is substantiated only when there is a concrete solution (13) in whole numbers. In order to check validity of (13) it is necessary to do the same discourse as before, since equations (7) and (13) are identical by their properties. This procedure can be continued to infinity in the direction of decreasing whole numbers under condition that sequence of chained equalities never stops, i.e., numbers x_0^2 and y_0^2 in (12) will be always whole. If it is not so, i.e., x_0^2 and y_0^2 in chained equalities (13) turn out to be fractions, then this means that solution (7) does not exist in the system of right-angled numbers**.** Actually, since all partitions of the type (12) are built from the very beginning exclusively on the set of right-angled numbers' squares being in fact whole items of finite series of partitions, then non-whole x_0^2 and y_0^2 show pointlessness of such procedure, i.e., the absence of integer solution (7) or zero solution. On the other hand, infinite sequence of chained equalities (13) leads to infinite decreasing of positive whole numbers that is impossible and therefore assuming that there exists an integer solution of (7) when $n > 2$ is not true. Thus the theorem is proved both for all even and for all odd degrees of whole numbers.

To visualize the arithmetic approach in the given proof, let us take Fig.1 from [4] showing fractal picture of similar right-angled triangles emerging from chains of proportions (8) during the quantum motion of the figure (when segments k and l change by unit) on the non-orientable surface. Instantaneous shot of this motion is represented in terms of Euclidean geometry when two independent diameters of small circles rotate synchronously in two opposite directions.



3. Conclusion. From arithmetic geometry of ancients to new paradigm of fractal world.

Attentive consideration of Fig.1 conduces to right comprehension of quantum paradigm in science. For example, if to lay two-dimentional Hilbert space on the picture of Fig.1, then it shows intrinsic transformations of space-time during the quantum jump of a quantum system from the state Φ_1 to the alternative state Φ_2 [4]. Such unknown quantum mechanics requires apparently new development in investigation of objective space-time [5-7]. These possibilities can be suggested by innovative trends in global science – progress of fractal geometry and geometrization of Poincare's Conjecture (now theorem) [6]. Previous achievements in treatment of physical reality and the laws of the Universe [8] must be revised from the point of view of generalized Fermat's Last Theorem, which is in fact Beal's Conjecture (theorem). Such modern unified approach to objective reality [4-7] allows to solve many chronic problems of mankind [9], for example, how to transform gravitational modality of energy into electromagnetic one and vice versa owing to changeable topological structure of space-time. On this way arithmetic algebraic geometry may play determinative role.

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Ю. А. Ивлиев Гипотеза Биля как глобальный прорыв в естественных науках.

Аннотация: Справедливость гипотезы Биля в математике установлена методами арифметической геометрии, известными еще в древности. Обнаруживается, что эта теорема играет существенную роль в познании объективного мира.

Ключевые слова: Гипотеза Биля, Великая теорема Ферма, фрактальная геометрия, квантовый подход, глобальная наука.

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